



Multi-scale fibre-based optical frequency combs:
science, technology and applications (MEFISTA)

Deliverables D2.4 (D11) MEFISTA

Modelocking and dual frequency comb lasers based on multimode fibres and resonators

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Analytical Demonstration of non-Hermitian Mode Coupling

Introduction

After numerically showing the principal possibility of the non-Hermitian mode coupling, which was demonstrated in deliverable D2.1 and D2.2, see also [1], we started an analytic approach to derive more rigorous theory calculating precisely the mode interactions as demonstrated in the previous deliverable D2.3. Therefore, from Deliverable D2.3 our chain of coupled mode equation is given by:

$$\frac{\partial a_m}{\partial z} = (i\beta_m a_m + imq a_m) + im_+ C_{m+1,m} a_{m+1} + im_- C_{m-1,m} a_{m-1} \quad (1)$$

where $m_+ = m_1 + im_2 e^{i\phi}$, $m_- = m_1 + im_2 e^{-i\phi}$, $\beta_m = -(2p + l + 1)\sqrt{2\Delta}/r_c$ and the coupling coefficient is given by: $C_{nm} = \iint F_n F_m^* e^{-\frac{r^2}{r_0^2}} r dr d\theta$ and in our case $C_{nm} = C_{mn}$. In order to simplify the analytics, we first remove the common phase shift and set the coupling coefficients equal:

$$\frac{\partial a_m}{\partial z} = im\delta a_m + im_+ C a_{m+1} + im_- C a_{m-1} \quad (2)$$

We first consider the three-mode case for which the chain of coupled mode equations is:

$$\begin{aligned} \dot{a}_1 &= i\delta a_1 + im_+ C a_2 \\ \dot{a}_2 &= im_- C a_1 + i2\delta a_2 + im_+ C a_3 \\ \dot{a}_3 &= im_- C a_2 + i3\delta a_3 \end{aligned} \quad (3)$$

We make a shift to have symmetric equations as:

$$\begin{aligned} \dot{a}_1 &= -i\delta a_1 + im_+ C a_2 \\ \dot{a}_2 &= im_- C a_1 + im_+ C a_3 \\ \dot{a}_3 &= im_- C a_2 + i\delta a_3 \end{aligned} \quad (4)$$

The eigenvalues of the system of equations (4) are given by:

$$\lambda = 0, \pm\sqrt{2C^2(m_2^2 - m_1^2) - 4iC^2 m_1 m_2 \cos(\phi) - \delta^2} \quad (5)$$

Results

In order to calculate the locked mode state, we first calculate the largest eigenvalue from the coupling matrix of the set of equations (4), for such three-mode case and the corresponding eigenvectors.

Figure 1a and 1b depicts how the locked mode varies with the spatial shift (ϕ) between the propagation constant and of the gain/loss coefficient and the modulation frequency (q), respectively.

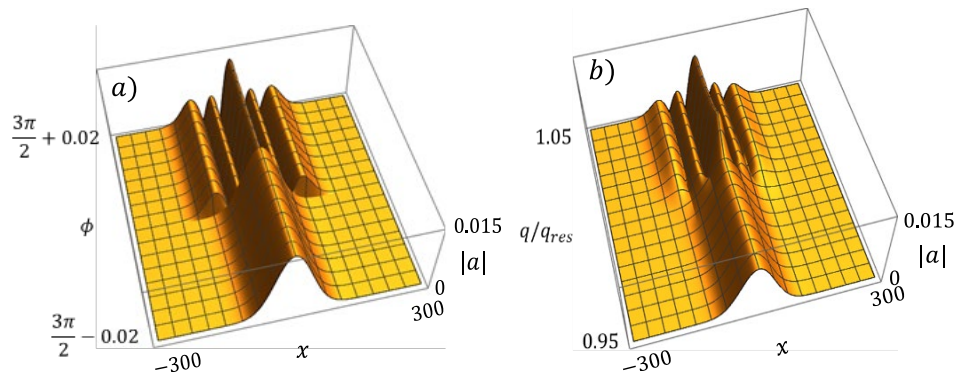


Fig 1: a) Variation of the locked mode with the spatial delay Φ and b) with modulation frequency q

Currently, we are exploring the more general cases considering more than three modes. In addition, the coupling coefficient values in practical are not constant and vary with the mode number as shown in figure 2. Therefore, we are considering this scenario too in our current calculations.

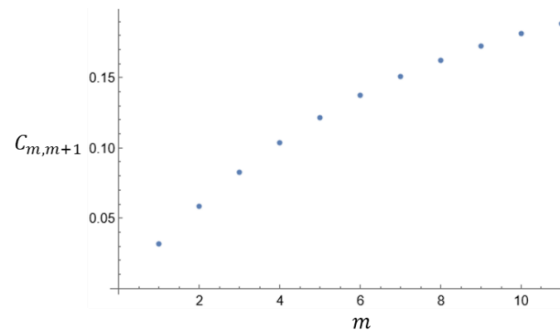


Fig 2: Variation of the coupling coefficient with the mode number

Conclusions

This analysis provides promising analytical results on the physical insight of the mode coupling. The article under this deliverable is presently in processing, with promising results deserving publication (more technical than conceptual).

Collaboration work with NKT Photonics

Introduction

In parallel with the above-mentioned work, we are also collaborating with one of our partners NKT Photonics to fulfill first secondment of the ESR 2. We plan to achieve numerical simulation of the temporal dynamics and frequency-domain envelope of dark solitons in normal-dispersion silicon-nitride microresonators. We are trying to demonstrate numerically the observed behavior of dark solitons in their experimental setup formed by two normal-dispersion silicon-nitride microresonators, where light is coupled between a main ring and an auxiliary one as shown in figure 3, [2].

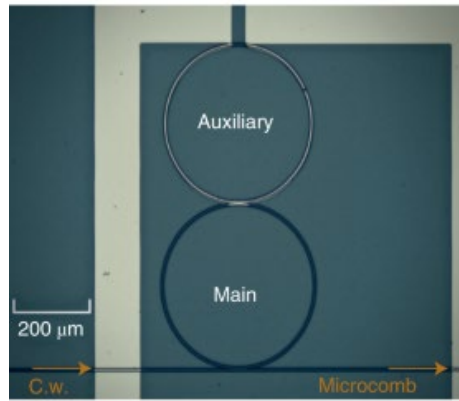


Fig 3: Dual ring resonator.

At first, we start to build our own numerical program considering only one ring without including the third order dispersion and the Raman response. We solve the standard Lugiato-Lefever equation (LLE) [3], written as:

$$t_R \frac{\partial A(t, \tau)}{\partial t} = \left[-\alpha - i\delta_0 - iL \frac{\beta_2}{2} \frac{\partial^2}{\partial \tau^2} + i\gamma L |A(T, \tau)|^2 \right] A(t, \tau) + \sqrt{\theta} E_{in} \quad (6)$$

Where, $A(t, \tau)$ is the intracavity field; t slow time; τ fast time; t_R cavity roundtrip time; α roundtrip amplitude loss; θ the bus waveguide coupling coefficient; $\delta_0 = (\omega_0 - \omega_p)t_R$ the phase detuning where ω_p is the pump frequency and ω_0 is the resonance frequency closest to ω_p ; L is the roundtrip length; β_2 the second-order dispersion coefficient; γ nonlinear Kerr coefficient; E_{in} external driving field (i.e., pump field).

In order to simulate the equation (6), first we normalize the equation according to [3]. The numerical simulation results are shown in figure 4. Figure 4a shows the temporal evolution of dark soliton. Figures 4b and 4c show the time domain amplitude of the stable dark soliton and the frequency domain comb amplitude respectively.

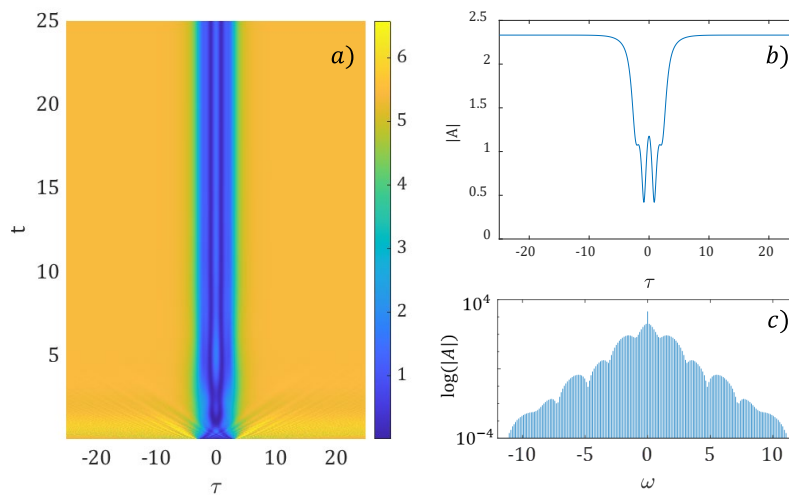


Fig 4: Numerical simulation of dark pulses in normal dispersion region.

In the next step, we will consider both the third order dispersion and the Raman response. Then, the modified equation (6) becomes [4]:

$$t_R \frac{\partial A(t, \tau)}{\partial t} = \left[-\alpha - i\delta_0 - iL \frac{\beta_2}{2} \frac{\partial^2}{\partial \tau^2} + L \frac{\beta_3}{2} \frac{\partial^3}{\partial \tau^3} + i\gamma L(1 - f_R)|A(T, \tau)|^2 + i\gamma L f_R (R * |A(T, \tau)|^2) \right] A(t, \tau) + \sqrt{\theta} E_{in} \quad (7)$$

where f_R is the fractional coefficient which determines the strength of the stimulated Raman scattering term, assumed to be 0.18 for silica and * denotes the convolution; $R(\tau)$ is the Raman response function:

$$R(\tau) = \frac{\tau_1^2 + \tau_2^2}{\tau_1 \tau_2} e^{-\tau/\tau_2} \sin(\tau/\tau_1)$$

where $\tau_1 = 12.2$ fs and $\tau_2 = 32$ fs for fused-silica based fibers.

Once the integration of the above model is performed for a one ring case, we plan to add the auxiliary ring to simulate and study the dual ring resonator.

Conclusions and perspectives:

From the preliminary results, this common work with NKT Photonics (ESR 6) and UPC (ESR 2) is expected to lead to promising results. It may help to understand the behavior of dark solitons in normal-dispersion silicon-nitride dual ring microresonators, and would eventually result in a collaborative publication. Regarding this work, the ESR 2 will be travelling to NKT Photonics in the month of March-April to fulfill his first secondment. After completing this secondment, he will be travelling to Aston University to complete his second secondment where he will be working on Development of novel mode locking techniques, based on non-Hermitian temporal modulation in active mode synchronization regimes – the combination of properly phase shifted amplitude and the phase modulation.

References

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