Multi-scale fibre-based optical frequency combs: science, technology and applications (MEFISTA)

Deliverables D2.3 (D10) MEFISTA

Modelocking and frequency combs with features on demand

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BENEFICIARIES

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Higher order Laguerre-Gaussian mode generation

In the previous deliverable, D2.2 we showed how the simultaneous, and spatially shifted, modulation of the propagation constant and of the gain/loss coefficient along the Graded INdex (GRIN) multimode fibers (MMFs) results in unidirectional coupling among the modes. In the case of the non-Hermitian coupling towards the lower order modes, such unidirectionality leads to the mode-cleaning effect for particular modulation parameter. Now, we here use the counter phenomena; i.e. coupling towards the higher order modes by changing the phase delay between the two modulations. This condition may be used to generate higher order modes from the lower order ones. Figures 1a and 1b show the generation of the even and odd Laguerre-Gauss (LG) order higher modes, respectively, along the propagation of the modulated GRIN MMF, as numerically calculated from the overlap integral of the field for the different modes.

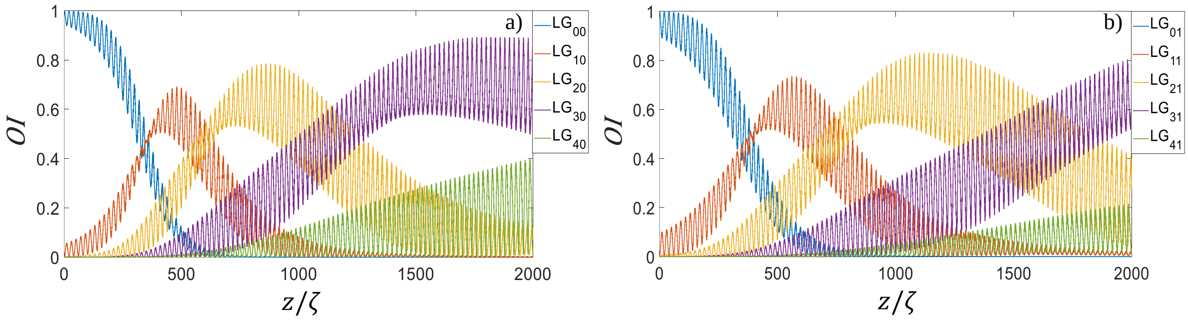

Fig 1: Higher order modes generation upon unidirectional mode coupling. Left: generation of even order LG modes. Right: Generation of odd order LG modes.

Analytical Demonstration of non-Hermitian Mode Coupling

Introduction

After numerically showing the principal possibility of the non-Hermitian mode coupling, which was demonstrated in deliverable D2.1 and D2.2, see also [1], we face in this next stage an analytic approach to derive more rigorous theory calculating precisely the mode interactions. We derive the chain of coupled equations including the nonreciprocal mode coupling.

We start with the linear Schrodinger equation in the presence of the non-Hermitian potential, [1]:

$$\frac{\partial A}{\partial z} = i\frac{1}{2}\nabla^2 A - i\frac{\Delta}{r_c^2}r^2 A + i\left[m_1\left(e^{iqz} + e^{-iqz}\right) + im_2\left(e^{iqz+i\phi} + e^{-iqz-i\phi}\right)\right]e^{-\frac{r^2}{r_0^2}}A$$

where $A(x, y, z)$ is the complex field amplitude envelope in the paraxial approximation evolving along z. The space coordinates are normalized to $k_0^{-1} = \lambda/2\pi$; where $k_0 = \omega_0 n_{co}/c$ is the light wavenumber, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the Laplacian in transverse space, r_c is the core radius, $\Delta = \left(n_{co}^2 - n_{cl}^2\right)/\left(2n_{co}^2\right)$ is the relative index difference, and n_{co} (n_{cl}) is the refractive index of the fiber core (cladding), respectively. We assume the solution of the filed in the system in the form of a summation of all modes present in the fiber as:

$$A = \sum_n a_n F_n(r) e^{-inqz}$$

which gives the chain of coupled mode equation as follows:

$$\frac{\partial a_m}{\partial z} = (i\beta_m a_m + imqa_m) + im_+ C_{m+1,m} a_{m+1} + im_- C_{m-1,m} a_{m-1}$$

where $m_+ = m_1 + im_2 e^{i\phi}$, $m_- = m_1 + im_2 e^{-i\phi}$, $\beta_m = -(2p + l + 1)\sqrt{2\Delta}/r_c$ and the coupling coefficient is given by: $C_{nm} = \iint F_n F_m^* \, e^{-\frac{r^2}{r_0^2}} \, r dr d\theta$ and in our case $C_{nm} = C_{mn}$. Figure 2 shows the variation of the coupling coefficient with the mode number.

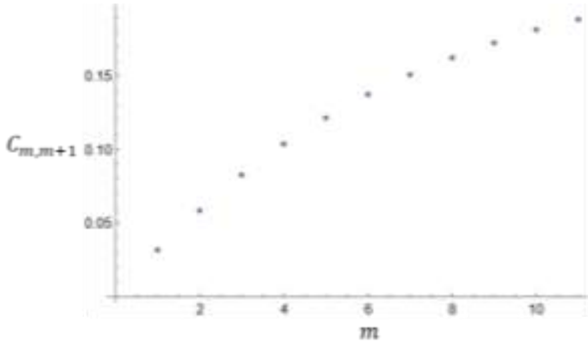

Fig 2: Variation of the coupling coefficient with the mode number

We express the infinite coupling matrix, asymmetric with respect to its diagonal, as:

$$\begin{pmatrix}
i(\beta_1 + q) & im_+C_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
im_-C_{12} & i(\beta_2 + 2q) & im_+C_{32} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & im_-C_{23} & i(\beta_3 + 3q) & im_+C_{43} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & im_-C_{34} & i(\beta_4 + 4q) & im_+C_{54} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & . & . & . & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & . & . & . & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & . & . & . & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & . & . & . & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . & . \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & i(\beta_n + nq)
\end{pmatrix}$$

whose eigenvalues determine the growth rates of the coupled mode states, and the corresponding eigenvectors define the mode composition in the locked mode state.
We start by considering the simpler case for five-mode coupling. In order to calculate the locked mode state we first calculate the largest eigenvalue from the above coupling matrix, for such five-mode case and the corresponding eigenvectors. Figure 3 depicts how the locked mode varies with the spatial shift between the propagation constant and of the gain/loss coefficient, as represented by the phase delay (ϕ) between the two modulations.

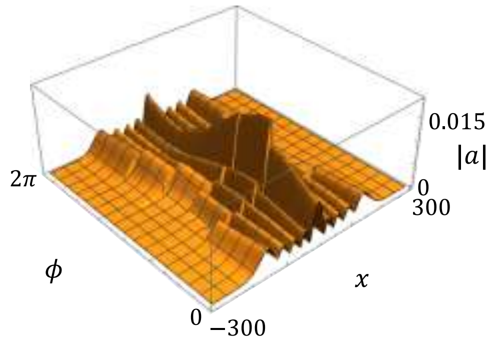

Fig 3: Variation of the locked mode with the spatial delay Φ

This analysis provides promising analytical results on the physical insight of the mode coupling. The article under this deliverable is presently in processing, with promising results deserving publication (more technical than conceptual).

Conclusions and perspectives:

Finally, after this deliverable is fully completed (and the article submitted), we foresee further steps towards the more realistic calculations of the specific schemes, involving nonlinearities (both, the Kerr and the gain-saturation nonlinearities). This will relate the conceptual theory developed in deliverables D2.1 and D2.2 to the real systems being fabricated and developed in the other WPs of the project MEFISTA, as well as with the other groups working on realization of frequency combs.

References

[1] M.N. Akhter, S.B. Ivars, M. Botey, R. Herrero, K. Staliunas, “Non-Hermitian Mode Cleaning in Periodically Modulated Multimode Fibers”, arXiv:2211.12762.

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