



# Multi-scale fibre-based optical frequency combs: science, technology and applications (MEFISTA)

## Deliverables D2.2 (D9) MEFISTA

### Passive modelocking due to non-Hermitian modulation of the potential

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## Consortium

### BENEFICIARIES



### PARTNERS ORGANISATIONS



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## Non-Hermitian Mode Cleaning

### Abstract

We show that the simultaneous modulation of the propagation constant and of the gain/loss coefficient along the Graded INDEX (GRIN) multimode fibers (MMFs) results in unidirectional coupling among the modes, which, depending on the modulation parameters, leads to the enhancement or reduction of the excitation of higher order transverse modes. In the latter case, effective mode-cleaning is predicted, in ideal case resulting in single-mode spatially coherent output. The effect is semi-analytically predicted on a simplified Gaussian beam approximation and numerically proven by solving the wave propagation equation introducing the modulated potential.

### Introduction

We here consider a harmonic non-Hermitian modulation of the propagation constant and the gain along a GRIN MMF with parabolic refraction index profile in transverse direction, as schematically illustrated in Fig.1a. The modulation of the propagation constant may be induced by a periodic variation of the refractive index, for instance, by doping the fiber core or modulating the fiber core radius. It is well known that such a modulation causes a symmetric coupling between the modes, as schematically indicated by the blue arrows in Fig.1c. In turn, we assume a modulation of the gain/loss profile of the MMF, with a particular spatial delay with respect to the Hermitian part of the modulation. The main idea is that the simultaneous action of both, the Hermitian and non-Hermitian parts of the modulation along the fiber may eventually induce a unidirectional coupling between the modes, as represented by the green arrows in Fig.1c. Here we mention the work in the previous deliverable (D2.1), where the principal possibility of non-Hermitian chaos control was proposed [1]. The unidirectional character of such coupling, and therefore the corresponding energy flow, may be either directed toward higher or lower index modes, depending on the shift between both modulations. For particular phase shifts, this is expected to eventually suppress the higher order modes, resulting in a lowest-mode coherent output [2]. This mode-cleaning mechanism is the basic aim of the present letter.

Here we substantize this idea: we show that the non-Hermitian modulation of the potential along the fiber indeed results in a tunable distribution of modes at the output. We first derive and explore a simplified model based on a Gaussian beam approximation, which predicts the effect, uncovering analytic insights, and provides estimations of the parameters. The proposal is then proven by direct numerical integration of the wave propagation equation along the fiber. On both models we analyse the modal energy distributions and, as the main result, we demonstrate a substantial condensation of radiation into the lowest order mode; resulting in a non-Hermitian mode-cleaning.

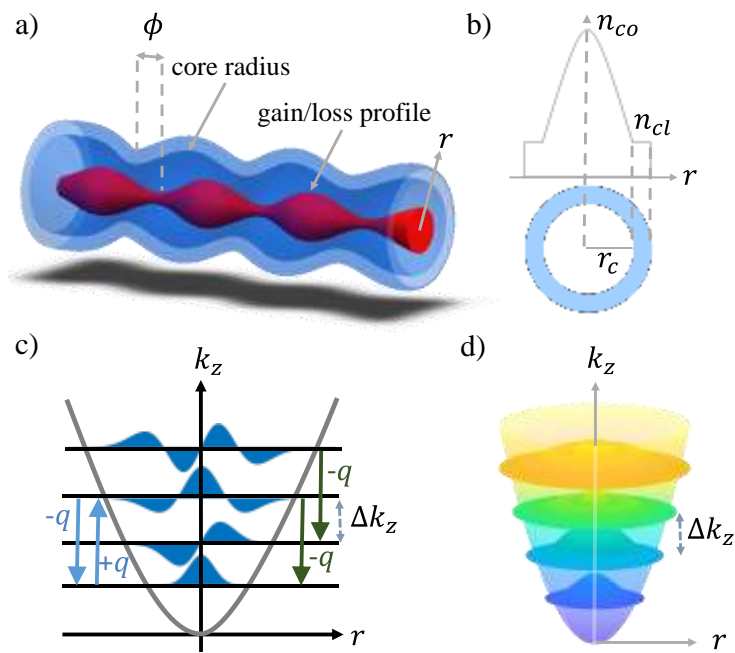


Figure 1. Modulated GRIN MMF.

## Results

The proof of the idea is summarized in Fig.2. The study is based on the Linear Schrodinger Equation (LSE) which is known as a universal model for light propagating in optical fibers. We demonstrate the effect by numerical integration of this model by considering the propagation of a noisy beam which can be considered as a combination of multiple modes. Note that the highly multimodal input distribution of the beam inside the MMF is gradually ‘attracted’ towards a bell-shaped transverse profile, as propagating along the fiber as shown in Fig. 2a. The insets of the Fig. 2b show the two dimensional transverse distribution of the beam at different propagation lengths. Figure 4b depicts the relative participation ( $O_{I_{pl}}$ ) of some Laguerre-Gauss modes,  $G_{pl}$ , with low mode order  $N$  ( $N = 2p + |l| + 1$ ), in the total field along the fiber. The participation of the lowest order mode ( $LG_{00}$ ) increases tending to 1 as the beam propagates along the fiber, while the participation of higher order modes decrease. Further to characterize the beam cleaning, we calculate the evolution of the beam width in real space, angular width in Fourier space (divergence), and the beam quality factor  $M^2$ . Figures Figs 2c-2e show a significant reduction of the beam width in both direct and wavenumber space, as the beam quality factor  $M^2$  rapidly approaches unity, acquiring an almost Gaussian beam profile.

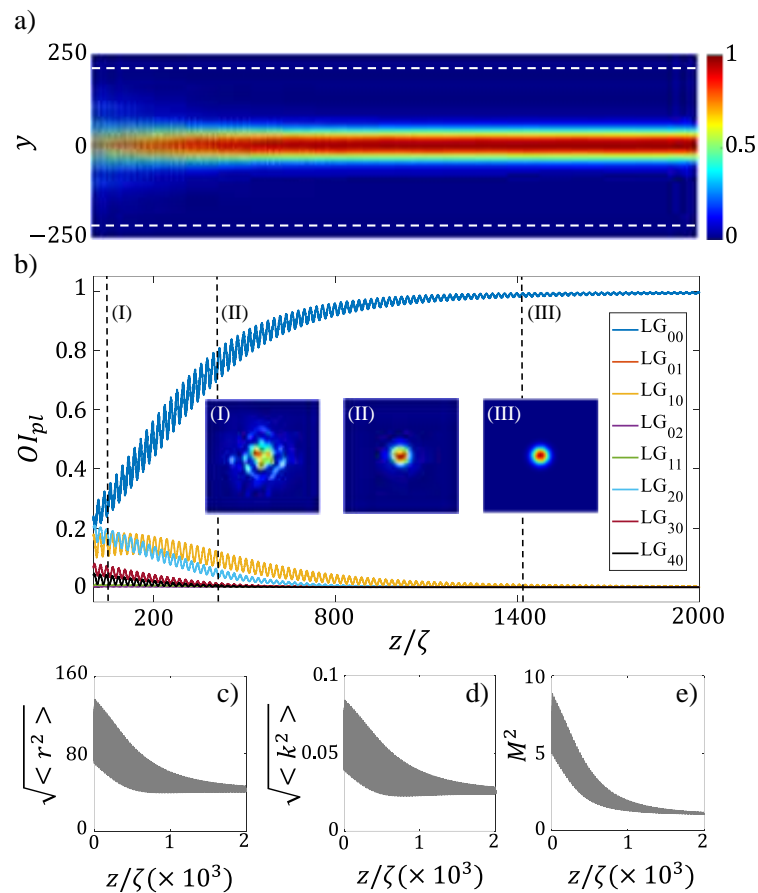


Figure 2. Evolution of the noisy beam along the modulated GRIN MMF.

## Conclusions

In conclusion, we propose and demonstrate a powerful mechanism for an effective spatial mode cleaning in GRIN MMFs. The proposal is based on the asymmetric mode coupling induced by the introduction of a non-Hermitian modulation of propagation constant and of the gain/loss coefficient along. The fiber may be modelled by a (2+1) D Schrödinger equation with a non-Hermitian potential, where the control over the coupling among transverse modes is mainly governed by the spatial shift between the real and imaginary parts of the complex potential. The results of the integration on the full model, provides a clear numerical proof of the proposal showing a significant mode-cleaning, irrespectively of the initial intensity. The demonstrated scheme could be experimentally realized within the current nanofabrication technologies, by modulating the core radius of an amplifying fiber of length on the order of meters with distributed losses.

The results under this deliverable has been submitted to Phys. Rev. Letters (the manuscript passed the first screening stage, and now is with the referees). The article is made open thorough the arXiv [3]. This, to our opinion is a conceptual work, showing the basics of the idea. The subsequent work will generalize the idea, extending the study to nonprecisely harmonic potential, to nonlinear cases, and to the cases with gain (the Raman or with coherent gain). The result of this deliverable will influence the other WPS of the project where the effect is to be demonstrated experimentally at least

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in its simplest implementation. In case of failure to realize such an experimental arrangement within the MEFISTA partners, we will also be searching for external collaborations.

This new mode locking mechanism, apart from its fundamental significance, can serve to improve the mode locking mechanisms in multimode fibers and, in collaboration with other WPs can initiate the generation of frequency combs in multimode/multicore fiber system. One of its major advantages is substantially enhanced frequency of the frequency combs (determined now by the intermode distance, and not by free spectral range of the whole fiber loop). This also opens a new possibility of synchronizing the frequency combs due to longitudinal modes (usual combs) and combs due to synchronized transverse modes (proposed in this delivery).

## References

- [1] S. B. Ivars, M. Botey, R. Herrero and K. Staliunas, *Chaos, Solitons & Fractals* 165, 112774 (2022).
- [2] M.N. Akhter, S.B. Ivars, M. Botey, R. Herrero, K. Staliunas, "Non-Hermitian Mode Cleaning in Periodically Modulated Multimode Fibers", *Physical Review Letters*, submitted 2022.
- [3] M.N. Akhter, S.B. Ivars, M. Botey, R. Herrero, K. Staliunas, "Non-Hermitian Mode Cleaning in Periodically Modulated Multimode Fibers", arXiv:2211.12762.



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